Post Modern Portfolio Theory (PMPT)

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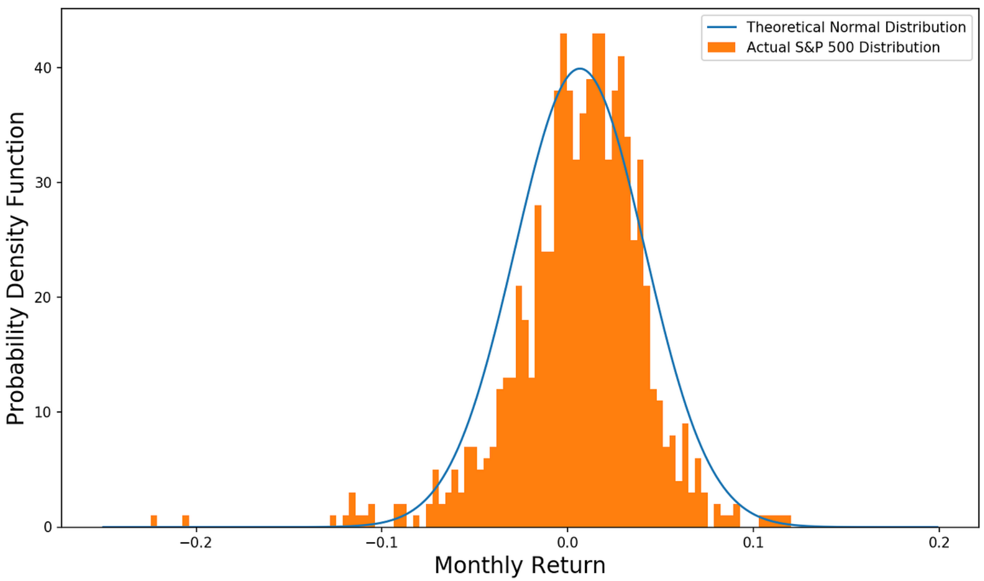
Theory: Limitations of MPT

Assumption #1: 모든 투자수익이 정규분포를 따른다는 가정 → **Lognormal Distribution**



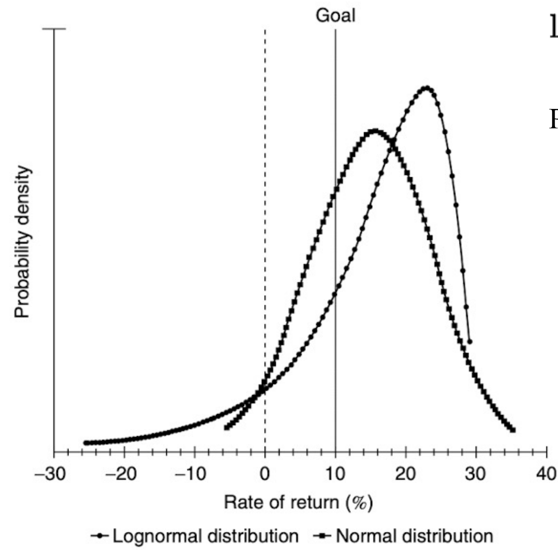


Figure 5.2 *Distribution of S&P 500 lognormal fit to monthly returns, 1992–96*

$$\ln(X) \sim \mathcal{N}(\mu, \sigma^2)$$

$$\mathrm{PDF} = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right)$$

Table 5.3 *Skewness statistics for five years, 1992–96*

Index	Volatility skewness	% of total variance from returns above the mean	% of total variance from returns below the mean	Statistical skewness*
Lehman aggregate	0.48	32.35	67.65	−0.18
Russell 2000	0.59	37.19	62.81	0.59
S&P 500	0.63	38.63	61.37	−0.28
90-day T-bill	0.93	48.26	51.74	−0.01
MSCI EAFE	1.21	54.67	45.33	0.13

*This is the usual statistical measure of skewness (the third moment of the distribution). Zero skewness represents symmetry while positive and negative values indicate positive and negative skewness, respectively.

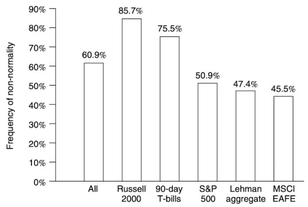


Figure 5.3 *Frequency of non-normal returns for major asset classes*

- 모든 투자수익이 정규분포를 따르지 않는다 (ex: S&P 500은 neg. skew)
- 5개 주요 index가 60% 확률로 정규분포를 따르지 않음 (Rom & Ferguson)
- PMPT는 투자수익에 더 적합한 fitting을 제공하는 **lognormal distribution** 활용 (3 parameter: μ, σ, τ – *extreme value*)

Source: Tony Yiu, Rom & Ferguson, YIG

Theory: Limitations of MPT

Assumption #2: risk = 변동성 (variance, standard deviation) → **MAR & Downside Risk (Downside Deviation)**

- “Risk = 변동성”으로 가정하는 MPT의 경우 평균에서 벗어난 모든 움직임이 리스크로 고려됨 → upside 변동성이 큰 것이 패널티로 적용
- PMPT는 “MAR(Minimum Acceptable Return) 아래로 생기는 변동성”을 리스크로 정의하며 실제 투자자에게 유의미한 Downside Risk 도출
- Downside Risk는 Downside Deviation 을 활용해 측정
- MAR을 조정해 투자자의 리스크 선호도 및 시장 전망 반영 가능 (bull market: higher MAR, bear market: lower MAR)

Source: Rom & Ferguson, Investing.com, YIG

Theory: Limitations of MPT

성과지표의 업그레이드: Sharpe ratio → Sortino ratio

MPT	PMPT
$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$	$$\text{Sortino Ratio} = \frac{R_p - r_f}{\sigma_d}$$
where: R_p = return of portfolio R_f = risk-free rate σ_p = standard deviation of the portfolio's excess return	**where:** R_p = Actual or expected portfolio return r_f = Risk-free rate σ_d = Standard deviation of the downside

Source: Investopedia, YIG

Implementation: Overview

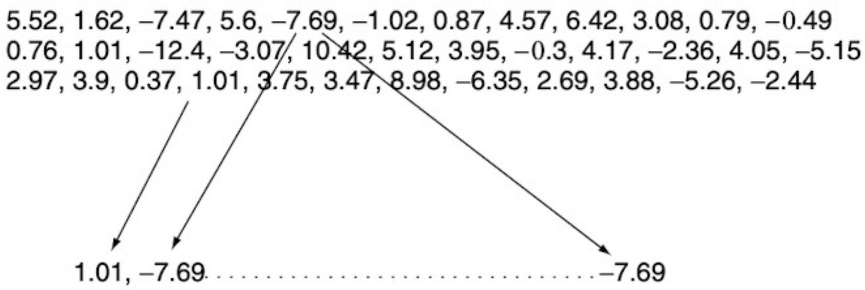
6-step Implementation Process

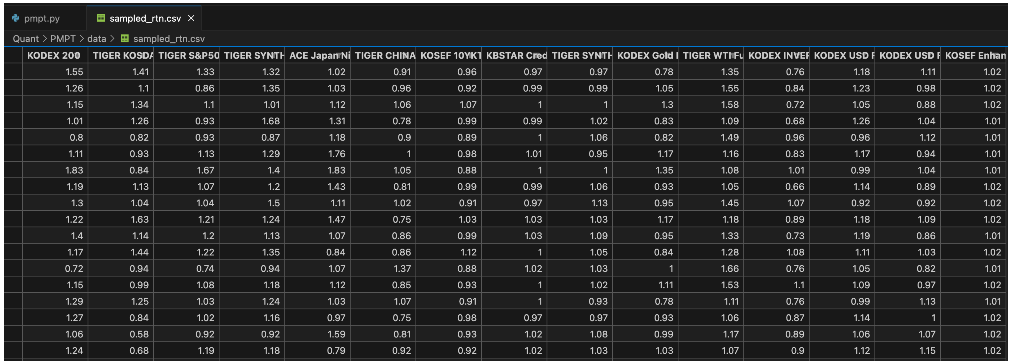
1. 포트폴리오 구성 자산 별 τ (extreme val.) 구하기 (optional)
2. 포트폴리오의 lognormal function 구하기
3. MAR 선정
4. Downside Deviation 구하기
5. Sortino ratio 구하기
6. Sortino ratio maximizing optimization을 통해 최적 포트폴리오 비중 도출

Source: YIG

Implementation #1: 포트폴리오 구성 자산 별 τ 구하기 (optional)

Random Sampling: Bootstrap Procedure

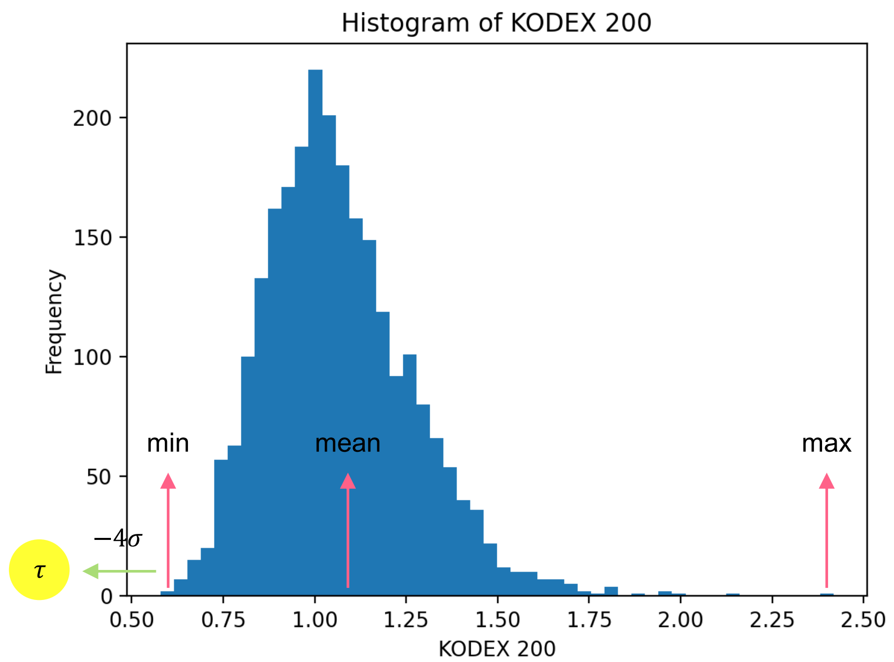




KODEX 200	TIGER KOSDA	TIGER S&P50	TIGER SYNTH	ACE JapanNi	TIGER CHINA	KOSEF 10YKT	KBSTAR Cred	TIGER SYNTH	KODEX Gold	TIGER WTIFu	KODEX INVEF	KODEX USD F	KODEX USD F	KOSEF Enhan
1.55	1.41	1.33	1.32	1.02	0.91	0.96	0.97	0.97	0.78	1.35	0.76	1.18	1.11	1.02
1.26	1.1	0.86	1.35	1.03	0.96	0.92	0.99	0.99	1.05	1.55	0.84	1.23	0.98	1.02
1.15	1.34	1.1	1.01	1.12	1.06	1.07	1	1	1.3	1.58	0.72	1.05	0.88	1.02
1.01	1.26	0.93	1.68	1.31	0.78	0.99	0.99	1.02	0.83	1.09	0.68	1.26	1.04	1.01
0.8	0.82	0.93	0.87	1.18	0.9	0.89	1	1.06	0.82	1.49	0.96	0.96	1.12	1.01
1.11	0.93	1.13	1.29	1.76	1	0.98	1.01	0.95	1.17	1.16	0.83	1.17	0.94	1.01
1.83	0.84	1.67	1.4	1.83	1.05	0.88	1	1	1.35	1.08	1.01	0.99	1.04	1.01
1.19	1.13	1.07	1.2	1.43	0.81	0.99	0.99	1.06	0.93	1.05	0.66	1.14	0.89	1.02
1.3	1.04	1.04	1.5	1.11	1.02	0.91	0.97	1.13	0.95	1.45	1.07	0.92	0.92	1.02
1.22	1.63	1.21	1.24	1.47	0.75	1.03	1.03	1.03	1.17	1.18	0.89	1.18	1.09	1.02
1.4	1.14	1.2	1.13	1.07	0.86	0.99	1.03	1.09	0.95	1.33	0.73	1.19	0.86	1.01
1.17	1.44	1.22	1.35	0.84	0.86	1.12	1	1.05	0.84	1.28	1.08	1.11	1.03	1.02
0.72	0.94	0.74	0.94	1.07	1.37	0.88	1.02	1.03	1	1.66	0.76	1.05	0.82	1.01
1.15	0.99	1.08	1.18	1.12	0.85	0.93	1	1.02	1.11	1.53	1.1	1.09	0.97	1.02
1.29	1.25	1.03	1.24	1.03	1.07	0.91	1	0.93	0.78	1.11	0.76	0.99	1.13	1.01
1.27	0.84	1.02	1.16	0.97	0.75	0.98	0.97	0.97	0.93	1.06	0.87	1.14	1	1.02
1.06	0.58	0.92	0.92	1.59	0.81	0.93	1.02	1.08	0.99	1.17	0.89	1.06	1.07	1.02
1.24	0.68	1.19	1.18	0.79	0.92	0.92	1.02	1.03	1.03	1.07	0.9	1.12	1.15	1.02

- Base data: 3 years, monthly returns data (3개년치 월 수익률)
- 랜덤으로 12개 월 수익률 sampling → compound → 랜덤 연 수익률 데이터 확보
- 위 random sampling 과정을 여러 번 (2,500 번) 반복해 large sample data 확보
- (optional) 시각화를 위해 히스토그램 도출

Simulation for attaining τ



- Sample data의 mean, stdev, min, max 계산
- Min, max 중 mean과 더 가까운 값 선정
- 해당 값을 mean으로부터 4 * stdev 만큼 멀리 이동 → τ
- Ex) mean = 12%, std = 8%, min = -15%, max = 70% → extreme value = -15% - 4(8%) = -47%

Source: Forsey, YIG

Implementation #2: 포트폴리오의 lognormal function 구하기

포트폴리오 mean, stdev, τ 구하기

Review: Portfolio Expected Return and Variance

- Notations: N assets, S possible states, $p(s)$ denotes the probability of state s, $r_i(s)$ denotes asset i's return in state s
- The portfolio weight for asset i: $w_i = \dfrac{\text{value of asset i}}{\text{value of all assets in portfolio}}$
- The portfolio return r_P in state s: $r_p(s) = w_1 r_1(s) + w_2 r_2(s) + \cdots + w_N r_N(s)$
- Expected return of a portfolio P: $E(r_P) = \sum_{s=1}^{S} p(s) r_P(s)$ or $E(r_P) = \sum_{i=1}^{N} w_i E(r_i)$
- Variance of a two-asset portfolio P: $\sigma_P^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B Cov(r_A, r_B)$

  or

  $\sigma_P^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \rho_{A,B}\sigma_A\sigma_B$

  Correlation

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Portfolio τ:

- Min = weighted combination of the min of all assets in portfolio
- Max = weighted combination of the max of all assets in portfolio

Lognormal function 공식

The three basic parameters estimated from the sample

Mean = sample mean
SD = sample standard deviation
τ = extreme value computed as described above

Some auxiliary parameters

$$\text{Dif} = |\text{Mean} - \tau|$$

$$\sigma = \ln\left(\left(\frac{\text{SD}}{\text{Dif}}\right)^2 + 1\right)$$

$$\mu = \ln(\text{Dif}) - \sigma^2$$

$$\alpha = \frac{1}{(\sqrt{2\pi} \cdot \sigma)}$$

$$\beta = -\frac{1}{(2\sigma^2)}$$

Formula for the lognormal curve $f(x)$

If the extreme value is a minimum and x is greater than the extreme value then

$$f(x) = \frac{\alpha}{x - \tau} \cdot \exp(\beta \cdot (\ln(x - \tau) - \mu))$$

If the extreme value is a maximum and x is less than the extreme value then

$$f(x) = \frac{\alpha}{\tau - x} \cdot \exp(\beta \cdot (\ln(\tau - x) - \mu))$$

Source: Forsey, YIG

Implementation #3, 4: MAR 선정, Downside Deviation 구하기

MAR Selection

MAR을 조정해 투자자의 리스크 선호도 및 시장 전망 반영 가능 (bull market: higher MAR, bear market: lower MAR)

- **투자자 리스크 선호도:** 우리의 경우 DB GAPS 상위권이 목표이기 때문에 top 20를 달성하기 위한 MAR 채택 가능 (현재 대략 5%)

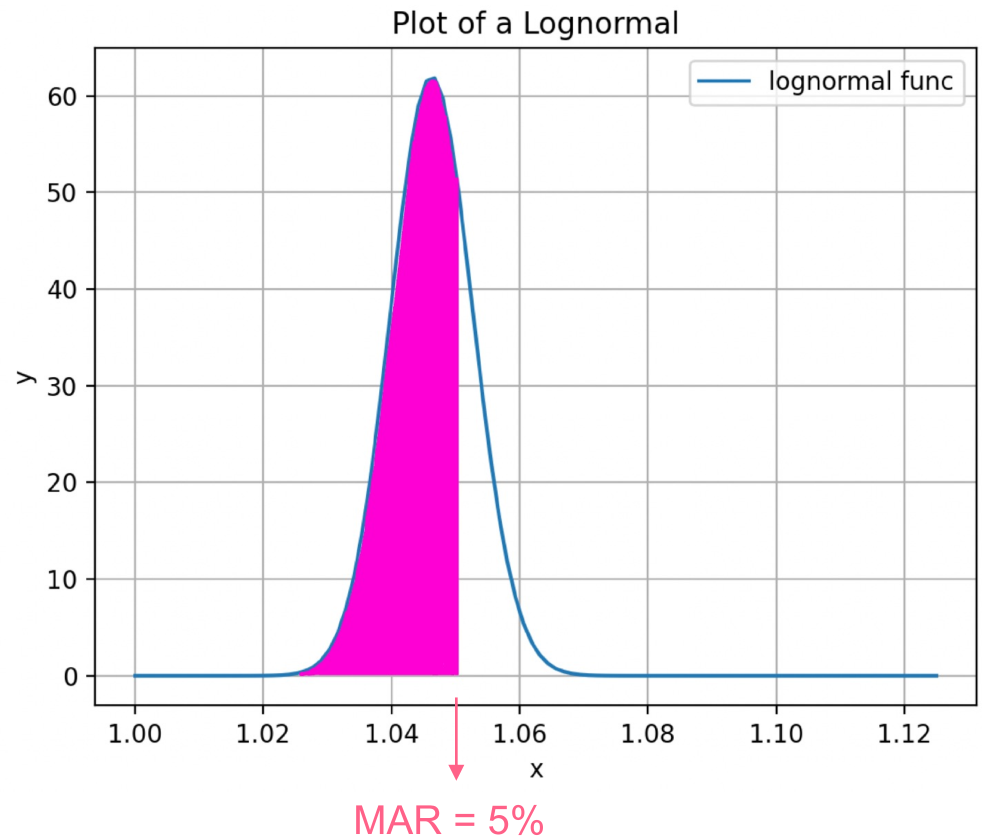
19	하락도 락이다	10,507,192,737	5.072
20	셋미나	10,507,034,220	5.070
21	전휘진	10,504,347,799	5.043

- **시장 전망:** 9월 FOMC 이후, 미국 기준금리 추가인상 가능성이 높아진 환경에서 bearish 한 시장 예상 → lower MAR

MAR = 5%

Downside Deviation Calculation

$$\text{Downside deviation} = \sqrt{\int_{-\infty}^{\text{MAR}} (\text{MAR} - x)^2 f(x)\, dx}$$



Source: Forsey, YIG

Implementation #5, 6:Sortino ratio 구하기, Optimization

Sortino Ratio Optimization for DB GAPS

```
 message: Optimization terminated successfully
 success: True
  status: 0
     fun: -10.731924791734984
       x: [-8.773e-10  1.885e-01 ... -8.302e-10 -8.668e-10]
     nit: 9
     jac: [ 7.073e+00  4.126e+00 ...  8.578e+00 -1.411e+00]
    nfev: 149
    njev: 9
[-8.77262712e-10  1.88495230e-01  5.18827967e-02 -9.38581817e-10
  4.81172030e-02 -1.04462193e-09 -9.43660172e-10 -3.87800395e-10
  2.00000001e-01 -9.66323143e-10  1.13009544e-01  1.88495230e-01
  2.00000002e-01 -8.30186443e-10 -8.66802225e-10]
```

Max Sortino Ratio

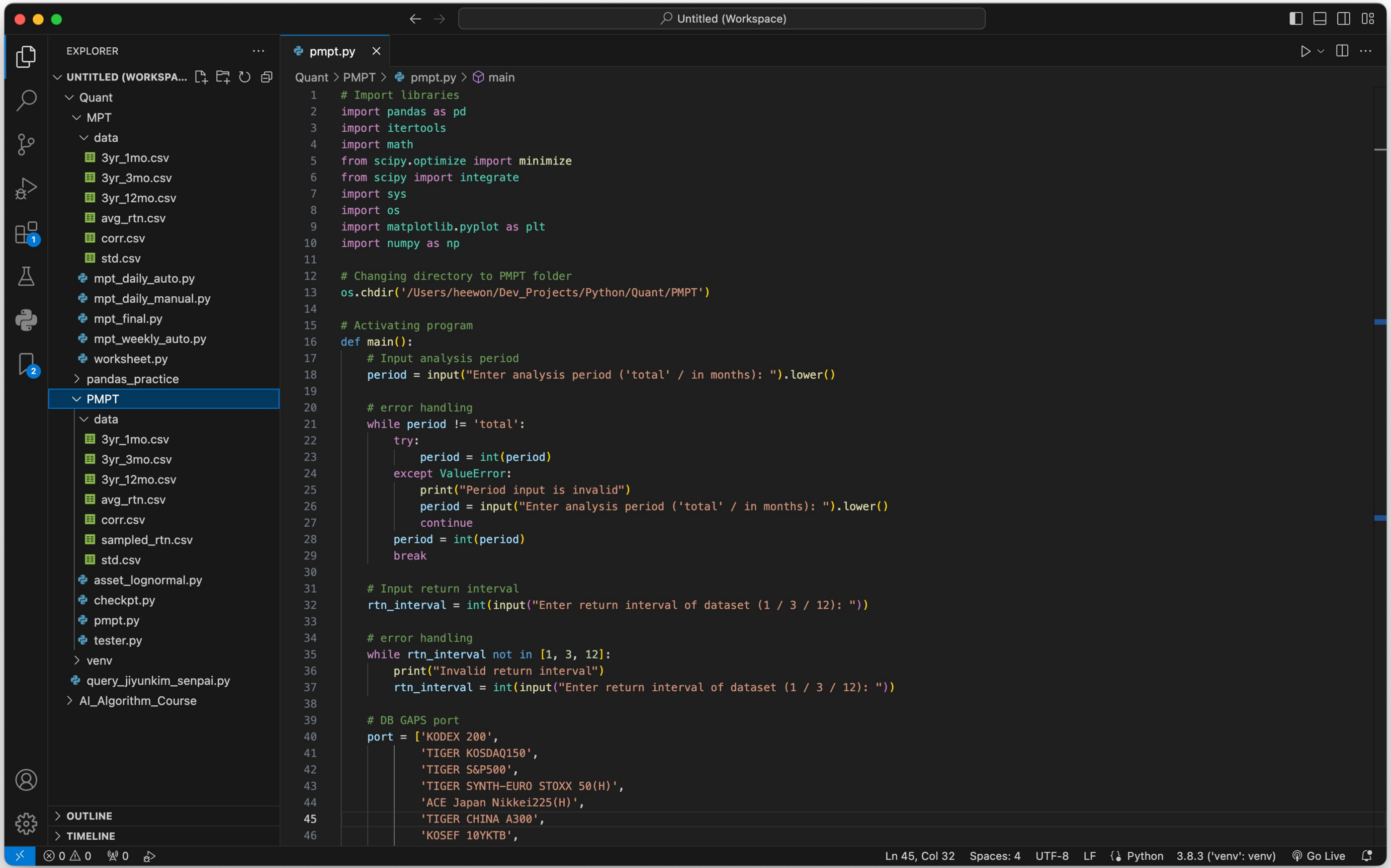
Number of iterations performed by optimizer

Optimal Portfolio Weight

['KODEX 200', 'TIGER KOSDAQ150', ... , 'KOSEF Enhanced Cash']

Source: YIG

Implementation: Python



Source: YIG

PMPT 결론

1. 포트폴리오 투자 수익률은 정규분포를 잘 따른다..! (특히 DB GAPS)
2. → 따라서 PMPT가 MPT 대비 갖는 edge가 크진 않았다
3. Random Sampling과 확률분포함수(PDF)를 사용하는 방법론이다 보니 Sample Size에 따라 결과값의 변동폭이 컸다
4. Lognormal로 특정 'market scenario'의 수익률 분포를 도출해 미래 시장 수익률을 전망하는 lognormal을 구할 수 있다

4.6 EXTENSION USING SCENARIOS

We know that next year's returns are dependent on economic and market forces that are changing. What can be done to include these changing conditions into our model? We briefly describe one approach, based on market scenarios. The idea is to divide past returns into a handful of groups based on the market scenario existing when they were generated. We then use our bootstrap approach to fit a lognormal curve to each asset for each scenario. Finally, we obtain a probability model for next year's return by using a mixture of these lognormal models with weights chosen according to our beliefs about next year's scenario.

Source: YIG